❸ Ratios, Graphs, Proportions and inverse proportions

contents

1 Percentages and *Buai* Change the following decimals into equivalent percentages and *Buai*.

(1) 0.2 (2) 0.62 (3) 3.2 (4) 0.164

2 Calculating percentages and *Buai* for various amounts Fill in ☐ with a suitable number.

(1) 60 yen is ☐ *wari* of 200 yen.

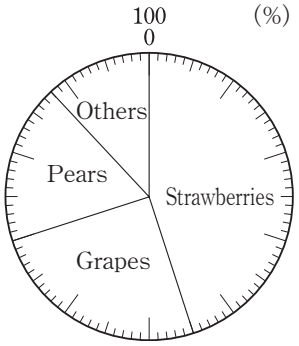
(2) 42 meters is ☐ % of 600 meters.

(3) 60% of 300 m^2 is ☐ m^2.

(4) 5 *wari* 4 *bu* of 600 g is ☐ g.

(5) The payment for an 800-yen article at a 40% discount is ☐ yen.

3 Proportions and graphs The circle graph on the right shows the land areas used for growing different fruits in an orchard. Answer the following questions.



(1) What percentage of the land area is used for growing strawberries, grapes and pears, respectively?

(2) If the whole area of the orchard is 7,200 m^2, how many square meters of land is used for growing strawberries, grapes and pears, respectively?

4 Ratios Answer the following questions.

(1) Simplify the following ratios.

① 8 : 24 ② 35 : 49 ③ 1.2 : 3 ④ $\frac{4}{3} : \frac{2}{5}$

(2) Fill in ☐ with a suitable number.

① 2 : 5=6 : ☐ ② ☐ : 12=7 : 4 ③ ☐ : 1.6=5 : 4

5 Proportions and inverse proportions y is proportional to x in the table (1), while y is inversely proportional to x in the table (2). Fill in blanks ① to ④.

(1) Proportion

x	2	4	②	8	④
y	8	①	24	③	40

(2) Inverse proportion

x	2	3	②	6	④
y	18	①	9	③	4

*1 *wari* is $\frac{1}{10}$ or 10%, 1 *bu* is $\frac{1}{100}$ or 1%, 1 *rin* is $\frac{1}{1{,}000}$ or 0.1%. These expressions are used in Japan and are called *buai*.

Chapter 2 Algebraic expressions 0

●Key points of study●

1 How to express products ☞P40～

(1) Expressions containing multiplication with variables* are written without × symbols. Numbers are always written in front of variables, and variables are usually arranged in alphabetic order.

(2) $a\times1$ and $1\times a$ are written as a, instead of $1a$. $(-1)\times a$ is written as $-a$.

(3) The product of a variable and itself is written using exponents: $a\times a\times a=a^3$

2 How to express quotients ☞P41～

(1) Expressions containing division with variables are written in fraction form without ÷ symbols.

(2) $\frac{a}{2}$ can also be written as $\frac{1}{2}a$: $a\div2=a\times\frac{1}{2}=\frac{1}{2}a=\frac{a}{2}$

(3) When the numerator or the denominator has a negative sign, it is put in front of the fraction: $\frac{-3}{x}=-\frac{3}{x}$

3 How to express quantities ☞P45～

When writing an expression that contains variables, first explain the relationships between the quantities in words. Then replace the words with variables and numbers to form a mathematical expression.

4 Evaluating expressions ☞P49～

(1) Plugging in a number for a variable in an expression is called substitution.

(2) Calculating the result of an expression by substituting a number for a variable is called evaluating the expression.

5 Terms and coefficients, Combining terms ☞P51～

(1) In an expression containing addition, each item joined by a + sign is called a term.

(2) In a term containing one or more variables, the number in front of the variable(s) is called the coefficient of the term.

(3) Terms containing the same variables are called "like terms" and can be combined into one term. In this way, an expression can be simplified.

6 Linear expressions ☞P51～

$3x-5$ consists of two terms: $3x$ and -5. Terms like $3x$ that contain only one variable are called terms of the first degree. An expression that contains only terms of the first degree or that indicates the sum of terms of the first degree and a number is called a linear expression.

7 Calculation with linear expressions ☞P53～

(1) To add or subtract two linear expressions, first remove the parentheses, then combine the terms having the same variable portion and those consisting of only numbers.

(2) To multiply a linear expression by a number, multiply each term of the expression by the number, using the distributive law as shown on the right.

$a(b+c)=ab+ac$

(3) To divide a linear expression by a number, divide each term of the expression by the number. You can also multiply each term by the reciprocal of the divisor.

8 How to express the relations of quantities ☞P59～

(1) An expression that uses an equal sign (=) to indicate an equivalent relationship between two quantities is called an equality.

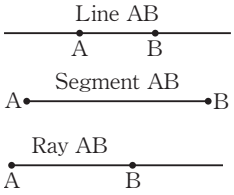
(2) An expression that uses an inequality sign ($<$, $>$, $\leqq$, $\geqq$) to indicate the relative size of two quantities is called an inequality.

* **Variables** are uppercase or lowercase letters used to represent unknown quantities in mathematic expressions.

Chapter 5 Plane figures

●Key points of study●

1 Lines and angles ☞P119～



(1) A straight line that passes through two points A and B is called line AB.

(2) A portion of line AB that has two endpoints A and B is called segment AB.

(3) A portion of line AB that starts at A and goes off in the direction of B to infinity is called ray AB.

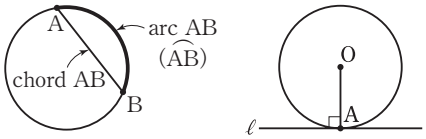
(4) The halfway point of a segment is called the midpoint.

(5) Triangle ABC is written as △ABC, using the symbol △.

(6) The angle formed by ray OA and ray OB is called angle AOB, which is written as ∠AOB.

(7) When lines AB and CD are parallel, the relationship is expressed as AB∥CD. When they are perpendicular, the relationship is expressed as AB⊥CD.

2 Circle arcs and chords, Tangents ☞P121～

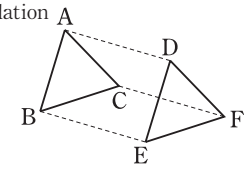


(1) When points A and B are on the circumference of a circle, the part from A to B is called arc AB, which is written as $\overset{\frown}{AB}$. The segment that links A and B is called chord AB.

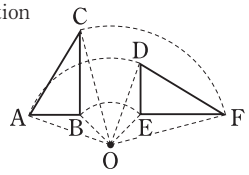
(2) When line ℓ touches circle O at a single point A, line ℓ is called a tangent of circle O and point A is called the tangent point. Line ℓ is perpendicular to OA ($\ell \perp$ OA).

3 Figure transformation ☞P124～

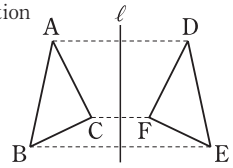
Translation



Rotation



Reflection

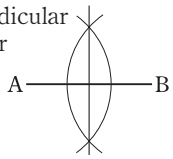


4 Figure construction ☞P128～

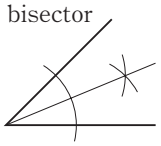
(1) Figure construction involves drawing figures with only a straightedge and compass. The straightedge is used to draw straight lines, and the compass is used to draw circles and replicate the length of segments.

(2) Since it is important to show the process of constructing figures, do not erase the lines or parts of circles that are drawn when solving construction problems.

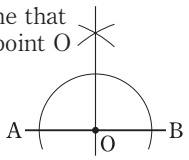
Perpendicular bisector



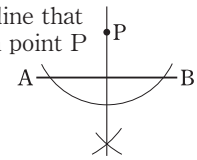
Angle bisector



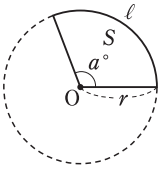
Perpendicular line that passes through point O



Perpendicular line that passes through point P



5 Sector arc length and area ☞P139～



(1) A portion of a circle's area bounded by two radii is called a sector.

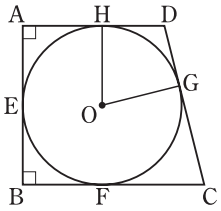
(2) The angle made by the two radii in the sector is called the central angle.

(3) For a sector having radius r, central angle $a°$, arc length ℓ, and area S, you can set up the following formulas.

① $\ell = 2\pi r \times \frac{a}{360}$ ② $S = \pi r^2 \times \frac{a}{360}$, $S = \frac{1}{2}\ell r$

Comprehension test for Chapter 5

1 **Parallel and perpendicular lines** Trapezoid ABCD is shown on the right. Points E, F, G, and H are the points where each side is tangent to circle O. Answer the following questions.



□(1) Write an expression to indicate that segments OG and OH have the same length.

□(2) Side AD is parallel to side BC. Express this relationship using a symbol.

□(3) Segment OG is perpendicular to side DC. Express this relationship using a symbol.

2 **Figure construction** Construct the following figures.

□(1) Perpendicular bisector of segment AB (Diagram 1) □(2) Angle bisector of ∠AOB (Diagram 2)

□(3) Straight line that passes through point P and is perpendicular to line ℓ (Diagram 3)

Diagram 1

Diagram 2

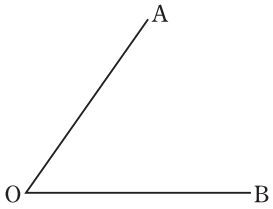
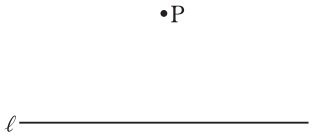
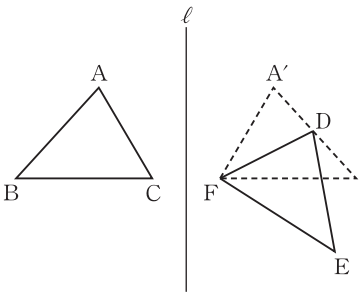


Diagram 3



3 **Figure transformation** As shown on the right, △DEF is the result when △ABC is reflected over the axis of line ℓ and then rotated 30° clockwise around the center of rotation F. Answer the following questions about △DEF and △ABC.



□(1) Which point corresponds to point A?

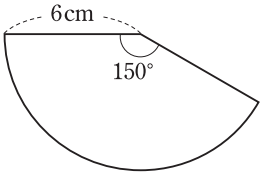
□(2) Which side corresponds to side AB?

□(3) Which angle corresponds to ∠DEF?

□(4) When ∠ACB=58°, find the size of ∠A′FE.

4 **Sectors** Find the arc length and area of each sector shown below.

□(1)



□(2)