Let's learn the basics 4 Using linear functions (4) ~Filling and draining water tanks~

Problem Figure 1 shows a cuboid-shaped tank that is 68 cm deep. Suppose this tank is filled with water by using Pipes A and B together for the first 12 minutes, and then, after that, Pipe B only until the tank is completely full. Figure 2 shows a graph of the relationship between x, the elapsed time in minutes measured from when the filling begins, and y, the depth of the water in the tank.

Figure 1

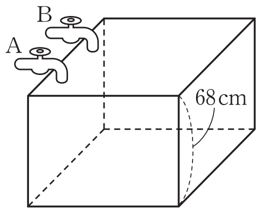
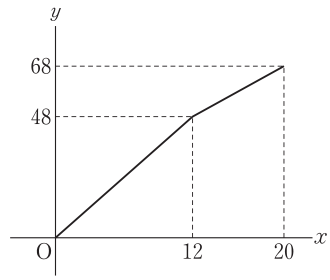


Figure 2



(1) When $12 \leqq x \leqq 20$, express y in terms of x.

(2) How many minutes after filling began did the water depth become 53 cm?

(3) If only Pipe A is used, how many cm per minute will the depth of the water increase?

Solution (1) Find the expression of the line that passes through two points (12, 48) and (20, 68). Expressing it as $y=ax+b$,
since $48=12a+b$ and $68=20a+b$, you get $a=2.5$ and $b=18$. Therefore, $y=2.5x+18$.

(2) According to the graph, $y=53$ when $12 \leqq x \leqq 20$.
Substituting $y=53$ in the expression you got in (1), you get $53=2.5x+18$, $x=14$.

(3) According to the graph, the rate of increase is $48 \div 12=4$ cm per minute when you use both pipes. Since it is 2.5 cm per minute when you use only Pipe B, it is $4-2.5=1.5$ cm when you use only Pipe A.

Answer (1) $y=2.5x+18$ (2) 14 minutes after (3) 1.5 cm

9 Figure 1 shows is a cuboid-shaped tank that is 43 cm deep. Suppose this tank is filled with water by using only Pipe A for the first 12 minutes, and then, after that, Pipes A and B together until the tank is completely full. Figure 2 shows a graph of the relationship between x, the elapsed time in munutes measured from when the filling begins, and y, the depth of the water in the tank.

Figure 1

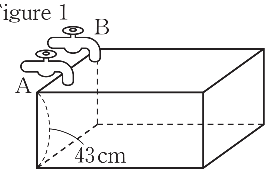
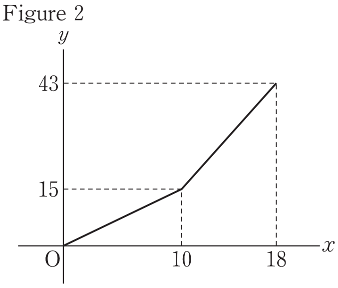


Figure 2



□(1) When $10 \leqq x \leqq 18$, express y in terms of x.

□(2) How many minutes after filling began did the water depth become 36 cm?

□(3) If only Pipe B is used, how many cm per minute will the depth of the water increase?

Let's learn the basics 2 Conditions for becoming special kinds of parallelograms

(1) Conditions for a parallelogram to be a rectangle

① One angle is a right angle.

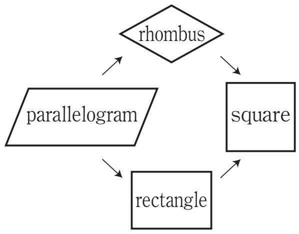
② The lengths of the diagonals are equal.

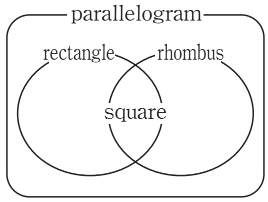
(2) Conditions for a parallelogram to become a rhombus

① A pair of adjoining sides are equal in length.

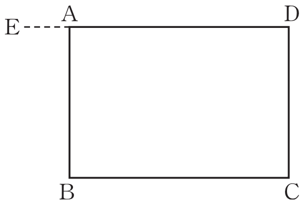
② The diagonals intersect perpendicularly.

(3) A parallelogram meeting the conditions for both a rectangle and a rhombus is called a square.



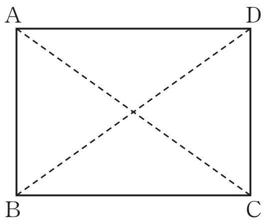


4 Prove that "A parallelogram with one right angle is a rectangle."



In ▱ABCD on the left, assume that ∠A is 90°.
When point E is on the extension of DA, since AB∥DC,
____=∠EAB=90°.
Since both pairs of opposite angles in a parallelogram are equal in size,
∠A=____=90° and ____=____=90°.
To sum up, ∠A=____=____=____=90°,
which means four angles are equal; therefore, ▱ABCD is a rectangle.

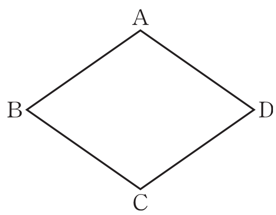
5 Prove that "A parallelogram whose diagonals are equal in length is a rectangle."



In ▱ABCD on the left, assume that AC is equal to BD in length.
In △ABC and △DCB,
as shown in the hypothesis, AC=____,
since both pairs of opposite sides in a parallelogram are equal in length, ____=DC,
and since it is shared by both triangles, ____=____.
The statements above mean that all three pairs of sides are equal in length, so
△ABC≡△DCB; therefore, ∠B=____.
Since both pairs of opposite angles in a parallelogram are equal in size,
∠A=∠C and ∠B=∠D,
and since the sum of interior angles in a quadrilateral is 360°, ____=____=____=____=90°, which means four angles are equal; therefore, ▱ABCD is a rectangle.

6 Prove that "A parallelogram having two adjoining sides that are equal in length is a rhombus."

In ▱ABCD on the right, assume that AB is equal to BC in length.
Since both pairs of opposite sides in a parallelogram are equal in length,
AB=CD and BC=DA.
Therefore, ____=____=____=____.
Since ____________ are equal, ▱ABCD is a rhombus.

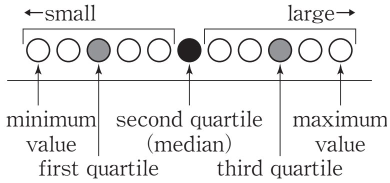


Chapter 7 Data Analysis

●Key points of study●

1 Quartile points and Box and Whisker Plot ☞P180

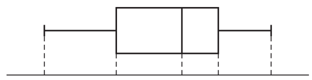
(1) **Quartile points**…When the values of a data set are arranged in order of size (from smallest to largest), three points can be defined that divide the data into four equal parts. The points are, from the smallest to the largest: **the first quartile, the second quartile** (median), and **the third quartile.**



(2) Interquartile range…The difference between the third quartile and the first quartile.

▶ **(Interquartile range) = (third quartile) − (first quartile)**

(3) **Box and whisker plot**…summarizes the distribution of the data using the 5 points: minimum value, first quartile, median, third quartile, and maximum value.



❶ Quartile points and Box and Whisker Plot ▶Exercises ⇨ P 184

Let's learn the basics 1 Quartile points (1)

When the data values are arranged from smallest to largest, the median of the left half of the data (including the minimum value) is **the first quartile**, the median of the right half of the data (including the maximum value) is **the third quartile**, the median of the entire data set is **the second quartile.**

Problem **The data on the right shows the time it takes (in minutes) for each of 11 students to get to school. Find the minimum value, the maximum value, and the quartile points of this data.**

26	36	27	50	48	27	40
34	32	23	33	(Unit : min.)		

Solution The data can be arranged from smallest to largest as follows.

23 26 (27) 27 32 (33) 34 36 (40) 48 50

minimum value	first quartile	second quartile	third quartile	maximum value
23	27	33	40	50

(Unit : min.)

Answer See the table above.

1 Find the minimum value, maximum value and quartile points of the following data.

□(1) 3 16 9 10 18 5 11 9 17 11 15 (Unit : hour)

□(2) 55 62 50 68 65 59 60 57 67 50 64 50 56 69 64 (Unit : g)

Comprehension test for Chapter 7

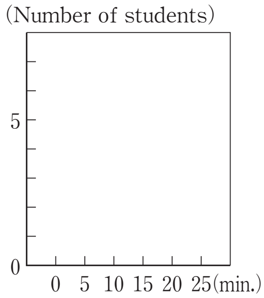
1 The following data shows the individual commuting times for a group of 15 students. Answer the following questions.

24	7	11	15	24	5	14	5
11	18	6	12	20	10	9	(Unit : min.)

□(1) Arrange the data in the frequency distribution chart on the right.

Class (min.)	Frequency (people)
at least less than	
0〜 5	
5〜10	
10〜15	
15〜20	
20〜25	
Total	15

□(2) Draw the corresponding histogram in the diagram on the right.

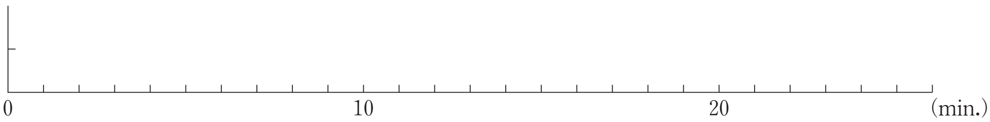


□(3) Fill in the following information in the table below: minimum value, maximum value, quartile points, interquartile range, and total range.

minimum value	first quartile	second quartile	third quartile	maximum value

interquartile range	range

□(4) Draw a box and whisker plot in the diagram below.



Application of Chapter 7

1 □ Box and whisker plots on the right summarize the scores achieved by basketball teams A, B, and C in their last 15 games. If you could select only one of these teams to play in the upcoming tournament, which team would you choose? Answer using letters A, B, or C, and explain the reasons for your choice.