Chapter 4 Functions

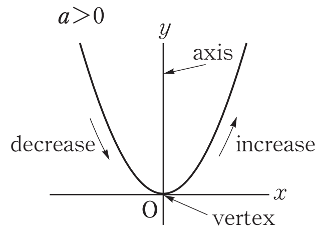
●Key points of study●

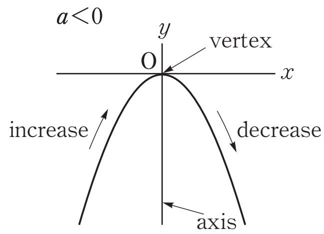
1 The function $y=ax^2$ ☞P81～

(1) When y is a function of x and the relationship between them is expressed as $y=ax^2$ (a is constant and $a \neq 0$), y is said to be proportional to the square of x.

(2) In the function $y=ax^2$, when you multiply the value of x by n, the corresponding value of y becomes n^2 times larger.

2 Graphing the function $y=ax^2$ ☞P83～





(1) The graph of function $y=ax^2$ is a smooth curve called a parabola.

(2) A parabola has an axis of symmetry (a.k.a., the axis of the parabola). The point where the parabola and its axis intersect is called the vertex of the parabola.

(3) When $a>0$,
- ① the graph opens upward.
- ② as the value of x increases, the value of y decreases within the range $x \leqq 0$ and increases within the range $x \geqq 0$.
- ③ $y \geqq 0$ regardless of the value of x.
- ④ and when $x=0$, the value of y is 0, which is its minimum value.

(4) When $a<0$,
- ① the graph opens downward.
- ② as the value of x increases, the value of y increases within the range $x \leqq 0$ and decreases within the range $x \geqq 0$.
- ③ $y \leqq 0$ regardless of the value of x.
- ④ and when $x=0$, the value of y is 0, which is its maximum value.

(5) As the absolute value of a increases, the graph opens more narrowly. The graph of $y=-ax^2$ is the reflection of the graph of $y=ax^2$ about the x-axis.

3 Rate of change ☞P88～

(1) When y is a function of x, the value of $\dfrac{\text{(increase in } y)}{\text{(increase in } x)}$ is called the rate of change.

"In the function $y=ax^2$, the rate of change as the value of x increases from p to q is $\dfrac{aq^2-ap^2}{q-p}$."

⇨This is equal to the slope of the straight line that passes through two points, whose respective x-coordinates are p and q, on the graph of function $y=ax^2$.

(2) The rate of change in the linear function $y=mx+n$ is constant, while that in the function $y=ax^2$ is not.

4 Point where the graphs of $y=ax^2$ and $y=mx+n$ intersect ☞P90～

① Solve the quadratic equation $ax^2=mx+n$ to find the value of the x-coordinate.

② Substitute the value of the x-coordinate into $y=ax^2$ or $y=mx+n$ to find the value of the y-coordinate.

● ● ● ● ● Exercises ● ● ● ● ● 2 Properties of the function $y=ax^2$

1 Answer the following questions.

□(1) Find the maximum and minimum values of y for each domain of x below for the function $y=-4x^2$.

① $-2 \leqq x \leqq -1$ ② $0 \leqq x \leqq 5$ ③ $-3 \leqq x \leqq 4$

□(2) The range of y is $b \leqq y \leqq 48$ when the domain of x is $-5 \leqq x \leqq 8$ for the function $y=ax^2$. Find the values of a and b.

2 Answer the following questions.

□(1) Find the rate of change as the value of x increases as follows for the function $y=\frac{3}{4}x^2$.

① from 1 to 3 ② from -8 to -4

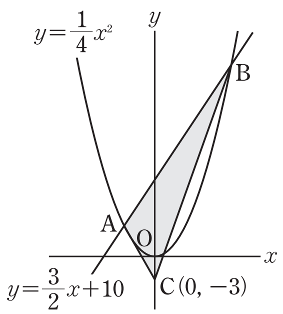
□(2) An object moving in a straight line travels y m in x seconds after it starts, and the relationship between x and y can be expressed as $y=4x^2$. Answer the following questions.

① What is the object's average speed (in meters per second) during the period between two and four seconds after it starts to move?

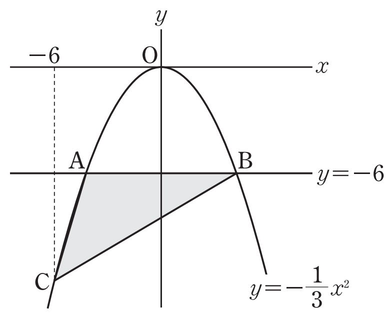
② When the average speed between t and $(t+2)$ seconds after it starts is 16 m per second, find the value of t.

3 In each figure below, find the coordinates of points A and B, where the parabola and the straight line intersect, and then find the area of △ABC.

□(1)



□(2)



4 Answer the following questions.

□(1) The graphs of the function $y=-2x^2$ and the linear function $y=ax-6$ intersect at points A and B. When the x-coordinate of A is -1, find the value of a and the coordinates of B.

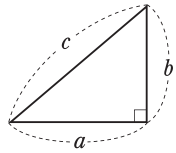
□(2) The graphs of the function $y=ax^2$ and the linear function $y=bx+12$ intersect at two points. When the x-coordinates of the points of intersection are -3 and 6, find the values of a and b.

Chapter 7 The Pythagorean theorem

●Key points of study●

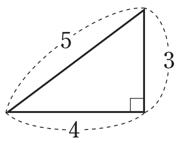
1 The Pythagorean theorem and the converse of the Pythagorean theorem ☞P167～

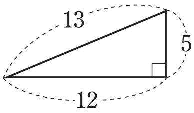
(1) When the lengths of two sides adjacent to the right angle in a right triangle are a and b and the length of the hypotenuse is c, the relationship between the lengths of the three sides is $a^2+b^2=c^2$. (the Pythagorean theorem)

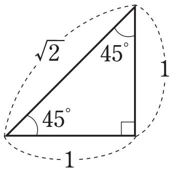


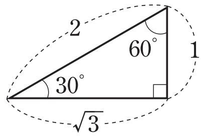
(2) When the lengths of the three sides of a triangle are a, b, and c and the relationship between them is $a^2+b^2=c^2$, the triangle is a right triangle in which the length of the hypotenuse is c. (the converse of the Pythagorean theorem)

2 Ratio of three sides in a special triangle ☞P171～





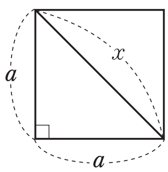




3 The Pythagorean theorem and plane figures ☞P171～

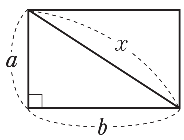
(1) Length of diagonals in a square

$x=\sqrt{2}\,a$



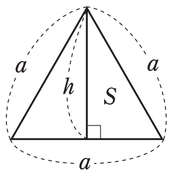
(2) Length of diagonals in a rectangle

$x=\sqrt{a^2+b^2}$



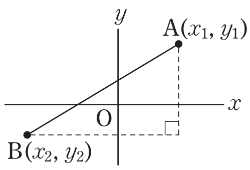
(3) Height and area of an equilateral triangle

$h=\dfrac{\sqrt{3}}{2}a,\ S=\dfrac{\sqrt{3}}{4}a^2$



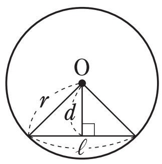
(4) Distance between two points

$\mathrm{AB}=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$



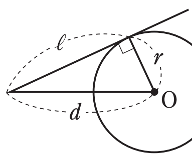
(5) Length of a chord in a circle

$\ell=2\sqrt{r^2-d^2}$



(6) Length of a tangent to a circle

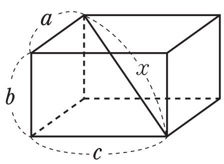
$\ell=\sqrt{d^2-r^2}$



4 The Pythagorean theorem and solids ☞P180～

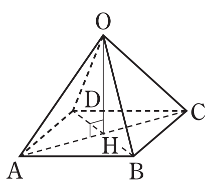
(1) Length of diagonals in a cuboid

$x=\sqrt{a^2+b^2+c^2}$



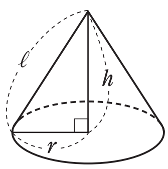
(2) Height of a square pyramid

$\mathrm{OH}=\sqrt{\mathrm{OA}^2-\mathrm{AH}^2}$

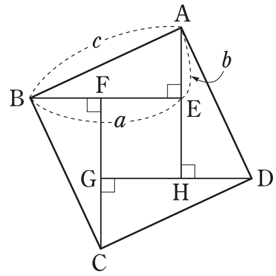


(3) Height of a cone

$h=\sqrt{\ell^2-r^2}$



Let's learn the basics **2** Proving that the Pythagorean theorem is true

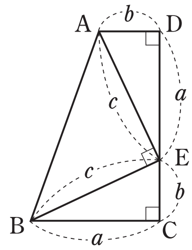


Problem **There are four congruent right triangles in square ABCD on the right. The lengths of the two sides adjacent to the right angle in each triangle are a and b and the length of each respective hypotenuse is c. Using this figure, prove that the Pythagorean theorem is true.**

Answer The total area of the four triangles is $\frac{1}{2}ab\times4=2ab$.

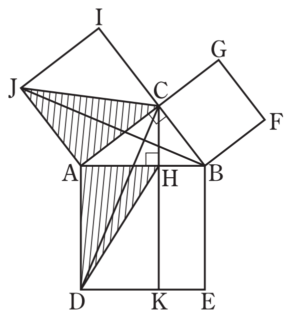
Quadrilateral EFGH is a square whose side length is $a-b$,
so its area is $(a-b)^2$.
The total area of the four triangles and square EFGH is
$2ab+(a-b)^2=2ab+(a^2-2ab+b^2)=a^2+b^2$. ……①
The side length of square ABCD is c, so its area is c^2. ……②
Since ① and ② are equal, it is true that $a^2+b^2=c^2$.

3 There are two congruent right triangles in trapezoid ABCD on the right. The lengths of the two sides adjacent to the right angle in each triangle are a and b and the length of each respective hypotenuse is c. When using this figure to prove that the Pythagorean theorem is true, fill in the blanks below to complete the proof.



〔Proof〕 The area of trapezoid ABCD is expressed in terms of a and b as $\boxed{\text{(i)}}$.
The total area of △ADE and △ECB is expressed in terms of a and b as $\boxed{\text{(ii)}}$.
The area of △ABE is expressed in terms of c as $\boxed{\text{(iii)}}$.
Since $\boxed{\text{(iv)}}-\boxed{\text{(v)}}=\frac{1}{2}c^2$,
it is true that $a^2+b^2=c^2$.

4 In the figure on the right, △ABC is a right triangle in which ∠ACB=90°, and each its side is shared by one of the following squares: ADEB, BFGC, ACIJ. The straight line drawn from C intersects AB perpendicularly at point H and intersects DE at point K. Fill in the blanks below to complete the proof that $AC^2+BC^2=AB^2$.



Since CJ is a diagonal of a square, square ACIJ=2△ACJ. …①
Since they share the same base and have the same height, △ACJ=$\boxed{\text{(i)}}$. …②
Since two pairs of sides and the angle between them are equal, △ABJ≡$\boxed{\text{(ii)}}$. …③
Since they share the same base and have the same height, △ADC=$\boxed{\text{(iii)}}$. …④
Since DH is a diagonal of a rectangle, rectangle ADKH=2$\boxed{\text{(iv)}}$. …⑤
It can be said from ① to ⑤ that square ACIJ=rectangle $\boxed{\text{(v)}}$. …⑥
Similarly, square BFGC=rectangle $\boxed{\text{(vi)}}$. …⑦
It can be said from ⑥ and ⑦ that square ACIJ+square BFGC=square $\boxed{\text{(vii)}}$.
Therefore, it is true that $AC^2+BC^2=AB^2$.